November 12, 2014

Conclusion of Personal Finance and Data

Goal

- to conclude personal finance and data management

Let's talk mortgages!

Most people do not have the cash to buy a home outright. They finance the home by obtaining a ***mortgage***. A mortgage is a loan to buy property, with the property acting as security. If the buyer, or **mortgagor**, fails to make the payments, the lending institution, or ***mortgagee***, can sell the property and use the money to pay off the mortgage.

The home buyer is required to have a percent of the house price as a **down payment**. The remainder is obtained through a mortgage. The buyer agrees to pay back the mortgage over a specified period of time with a series of equal regular payments. A mortgage, therefore, is an ***annuity***.

Mortgage payments are usually made monthly but can be paid more often. Since a large sum of money is involved, the time taken to repay a mortgage usually ranges from 10 to 30 years.

Mortgages in Canada cannot legally be compounded more frequently than semi-annually. For simplicity, we will consider both monthly compounding and monthly payments only.

Example # 1:

A mortgage of $190 000 is required to purchase a house. The mortgage will be repaid with equal monthly payments over 25 years at 8%/a. compounded monthly.

a) What is the monthly payment?
b) What is the total interest paid over the 25 years?

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

Present Value

Regular Payment

$$190000 = \frac{R\left[1-(1.00666666)^{-300}\right]}{0.00666666}$$

$$190000 = R(129.5646116)$$

$$\frac{190000}{129.5646116} = R$$

$$R = 1466.45$$

1466.45 x 300

= 439 934. 94

interest

439934.94 - 190000 = 249 934. 94

What's different about other annuities?

$$FV = \frac{R\left[(1+i)^{n}-1\right]}{i}$$

Example # 3:

Annika deposited $800 in an account at the end of every 6 months for 7 years. The account paid 9%/a. compounded semi-annually. Determine the amount in the account on the date of the last deposit.

FV = ?

R = $800

$i = \frac{0.09}{2}$

$= 0.045$

$n = 7 \times 2$

$= 14$

$$FV = \frac{R\left[(1+i)^{n}-1\right]}{i}$$

$$FV = \frac{800\left[(1.045)^{14}-1\right]}{0.045}$$

$$FV = 15145.69$$

Introduction to MATHEMATICAL MODELS

Goals
1. to be able to rearrange formulas
2. to be able to use exponent rules

Rearranging Formulas

When we solve the following question, what acronym do we use?

7*5+9(6+4)-1 BEDMAS

When we rearrange a formula, we use BEDMAS backwards!

SAMDEB

Examples

V = I R	I: $V = IR \rightarrow \frac{V}{R} = I$	R: $V = IR \rightarrow \frac{V}{I} = R$
$E=mc^2$ $c^2 = \frac{E}{m}$ $c = \sqrt{\frac{E}{m}}$	m: $m = \frac{E}{c^2}$	c: $\sqrt{c^2} = \sqrt{\frac{E}{m}}$ $c = \sqrt{\frac{E}{m}}$
P=5T-M	T: $M+P = 5t$ T: $\frac{M+P}{5} = t$	$M = 5t - P$

Let's try some harder ones:

$C = \frac{5(F-32)}{9}$ $\quad 9C = 5(F-32)$ $\quad \frac{9C}{5} = F-32$ $\quad \frac{9C}{5} + 32 = F$

$V = \frac{4}{3}\pi r^3$ $\quad 3V = 4\pi r^3$ $\quad \frac{3V}{4\pi} = r^3$ $\quad \sqrt[3]{\frac{3V}{4\pi}} = r$

1) $E = \frac{mv^2}{2}$

2) $A = P + I$

3) $E = mgh$ $\quad E = mgh$ $\quad \frac{E}{mg} = h$ $\quad \frac{E}{m} = gh$ $\quad \frac{E}{g} = h$

4) $S = \frac{d}{t}$

5) $V + y = \frac{w+d}{2}$ $\quad 2(v+y) = w+d$ $\quad 2(v+y) - w = d$

Exponent Rules

Multiplication

$x^d \cdot x^b = x^{d+b}$

Division

$x^c \div x^b = x^{c-b}$

Zero

$x^0 = 1$

Power of Power

$(x^e)^g = x^{e \times g}$

Negative

$x^{-c} = \frac{1}{x^c}$ $\left(\frac{x}{y}\right)^{-c} = \left(\frac{y}{x}\right)^{c}$

ALWAYS LEAVE YOUR EXPONENTS +

Power

Side Note

Cb^e

base
coefficient
exponent

$5^2 = 25$ ÷5
$5^1 = 5$ ÷5
$5^0 = 1$ ÷5
$5^{-1} = \frac{1}{5}$ ÷5
$5^{-2} = \frac{1}{5^2}$

State the base and the exponent of the following:

1. 5^3
2. 10^7
3. x^5
4. t^2

Write the following in exponential form:

5. $3 \times 3 \times 3 \times 3 \times 3 = 3^5$
6. $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$
7. $10 \times 10 \times 10 = 10^3$
8. $6 \times 6 \times 6 \times 6 = 6^4$
9. $(m)(m)(m)(m)(m)(m)(m) = m^7$

Write each number as a power of 10.

10. 100 10^2
11. 1000 10^3
12. 100 000 10^5
13. 10 000 10^4
14. 100 000 000 10^8
15. 10 000 000 10^7

Write as a power of 2.

16. 16 2^4
17. 64 2^6
18. 256 2^8
19. 2048 2^{11}
20. 512 2^9

Simplify the following exponential expressions:

1. $5^3 \times 5^4 = 5^{3+4} = 5^7$
2. $2^3 \times 2^7 = 2^{3+7} = 2^{10}$
3. $10^6 \times 10 = 10^7$
4. $y^2 \times y^4 = y^6$
5. $a \times a^6 = a^7$
6. $(2^3)^8 = 2^{3 \times 8} = 2^{24}$
7. $(3^5)^2 = 3^{10}$
8. $(4^2)^7 = 4^{2 \times 7} = 4^{14}$
9. $(y^3)^3 = y^9$
10. $(m^2)^{-5} = m^{-10} = \frac{1}{m^{10}}$

Homework

p. 362 # 1 - 6, 8, 12, 13